Enrollment No: ______________________ Exam Seat No: ______________________

C.U.SHAH UNIVERSITY

Summer Examination-2018

Subject Name : Advanced Calculus

Subject Code : 4SC03MTC1/4SC03ADC1 **Branch: B.Sc.(Physics)**

Semester : 3 **Date : 22/03/2018** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

1. Use of Programmable calculator & any other electronic instrument is prohibited.
2. Instructions written on main answer book are strictly to be obeyed.
3. Draw neat diagrams and figures (if necessary) at right places.
4. Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Verify Euler's theorem for $u = ax^2 + 2hxy + by^2$. **(02)**

b) If $x^4 + y^4 = 4b^2xy$, find $\frac{dy}{dx}$. **(02)**

c) Find interval on which the function $x^3 - 12x - 5$ is increasing or decreasing **(02)**

d) If $y = p\cos\theta, z = p\sin\theta$ then what is the value of $\frac{\partial p}{\partial y}$. **(02)**

e) Prove that $\beta(m,n) = \beta(m,n+1) + \beta(m+1,n)$. **(02)**

f) Find asymptotes of the curve $x^2y^2 = a^2(x^2 + y^2)$. parallel to co ordinate axis. **(02)**

g) Write the relation between Beta and Gama function **(01)**

h) Is the function $f(x,y) = \sin(\frac{x-y}{x+y})$ homogeneous? **(01)**

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) If the function $z = f(x,y)$ possesses the first order derivative in the domain D then prove that $dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$. **(08)**

b) If $u = \log(\tan x + \tan y + \tan z)$ then prove that $\sin 2x\frac{\partial u}{\partial x} + \sin 2y\frac{\partial u}{\partial y} + \sin 2z\frac{\partial u}{\partial z} = 2$. **(06)**

Q-3 **Attempt all questions** **(14)**

a) If $u = x + y + z, uv = y + z, uvw = z$ then prove that $\frac{\partial(u,v,w)}{\partial(x,y,z)}\frac{\partial(x,y,z)}{\partial(u,v,w)} = 1$. **(08)**

b) If $u = f\left(\frac{x}{y},\frac{y}{z},\frac{z}{x}\right)$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$. **(06)**

Q-4 **Attempt all questions** **(14)**

a) State and prove Taylor's theorem for the function of two variables. **(08)**

b) Evaluate: $\int_0^1 x^m (\log \frac{1}{x})^n \; dx.$ **(06)**

Q-5 **Attempt all questions** **(14)**

a) State and prove Duplication formula. **(08)**

b) Find extreme value of $f(x, y) = x^3 + y^3 - 3\,xy$ **(06)**

Q-6 **Attempt all questions** **(14)**

a) Find all asymptotes of the curve $x^3 + y^3 - 3axy = 0.$ **(05)**

b) Find the maximum value of $f(x, y, z) = xyz$ subject to the constraint $2x + 2y + z = 108$ using Lagrange's method of undetermined multipliers **(05)**

c) Evaluate: $\int_0^1 \frac{x^5}{\sqrt{1-x^4}} \; dx$ **(04)**

Q-7 **Attempt all questions** **(14)**

a) Find range of values of x for which the curve $y = x^4 - 6x^3 + 12x^2 + 5x + 7$ is concave upward and downward. Also find points of inflection in each case. **(05)**

b) Expand $e^x \cos y$ in powers of x and y up to three degree. **(05)**

c) Evaluate : $\int_0^1 \sqrt{x}\, \sqrt[3]{(1 - x^2)} \; dx$,with the help of beta function. **(04)**

Q-8 **Attempt all questions** **(14)**

a) Using definition of limit prove that $\lim_{(x,y)\to(1,3)} 5x + 7y = 26.$ **(05)**

b) If $x^3 + y^3 = 3ax^2$, prove that $\frac{d^2y}{dx^2} = -\frac{2a^2x^2}{y^5}.$ **(05)**

c) If $u = \sin^{-1}(x^3 + y^3)^{2/5}$, then prove that **(04)**

$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{6}{5}\tan u[\frac{6}{5}\sec^2 u - 1].$